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Research paper

Tooth contact analysis of crossed beveloid gear transmission with parabolic modification

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ABSTRACT

Considering the gear manufacturing and meshing theory, a numerical design approach with parabolic modifications was proposed to improve the mesh behaviors of crossed beveloid gear transmission. The generating procedure with parabolic modifications for the crossed beveloid gears with small shaft angle was proposed. And the analytical mesh model with misalignments of crossed beveloid gear transmission was developed. Then the effects of parabolic modifications on contact path and contact ratio were investigated. Also, the misalignments sensitivity was analyzed for the beveloid gears with and without parabolic modifications. Results show that the design situation with a positive parabola coefficient for the pinion and a negative parabola coefficient for the gear is better than other combinations of the parabola coefficients. Both the offset error and shaft angle error tend to deviate the contact path from the theoretical middle region. The parabolic modifications can reduce the sensitivity of contact path to the offset error and shaft angle error for crossed beveloid gear transmission effectively. Both axial pinion and gear position error has little influence on the mesh behaviors for the crossed beveloid gear pair with and without parabolic modifications.

1. Introduction

Beveloid gear which can realize the spatial transmission with small shaft angles is widely used in the fields of speedboat [1], all-wheel drive vehicles [2], radar tracking system and aviation precision machinery. It has many advantages including small volume, compact structure, high precision, easy manufacturing and low cost. However, it is theoretical point contact for the engagement and the higher resulted high sliding speed between beveloid gears can produce the early wear of gear tooth, which leads to the low bearing capacity of tooth surface and short service life.

Recently, lots of research has been performed on the geometry design and tooth contact analysis of beveloid gears. Tsai and Wu [3,4] discussed the geometry design method of beveloid gears and conducted the loaded tooth contact analysis (LTCA) of skew conical involute gear pair in approximate line contact. Brecher et al. [5] developed two methods to calculate the tooth load carrying capacity of beveloid gears with parallel axes. Eberhard et al. [6,7] developed a mesh model with the fully elastic multibody and investigated the effects of standard and non-standard tooth profiles on the contact characteristics of beveloid gears considering deformations. Liu et al. [8,9] presented two mathematical models of concave beveloid gears and performed the contact analysis of concave beveloid gear pair to investigate the contact behaviors of concave conical involute gear pair with non-parallel axes. Li et al [10] developed a new way to calculate the profile and axial errors of

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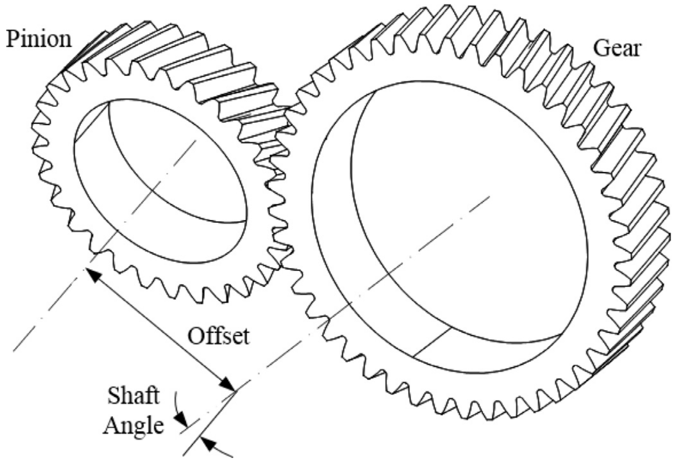


Fig. 1. Crossed beveloid gear transmission.

noninvolute beveloid gears. Zhu et al. [11,12] investigated the approximate line contact condition by controlling the angle between the first principal directions and analyzed the effects of the design parameters, load and misalignments on the mesh for crossed and intersected beveloid gears with small shaft angle. Brecher et al [13,14] simulated the manufacturing of beveloid gears and investigated the effect of manufacturing methods on the running behavior of beveloid gears. Brauer [15,16] derived the mathematical model of conical involute gears and researched the transmission error of anti-backlash conical involute gear transmission using a global-local finite element method. For the parabolic modification, Oswald et al. [17] discussed the effect of parabolic profile modification on the dynamic response of low-contact-ratio spur gears. Zhang et al. [18] modified the line segment of rack cutter into parabolic and investigated the effects of parabolic modifications on the transmission error and load distribution of helix gear transmission. Litvin [19] used the rack cutter with parabolic profile to generate the profile-crowned helical gear and analyzed the contact characteristics. However, the research studies mentioned above are mainly focused on geometric design and contact analysis of beveloid gears or parabolic modification on spur or helix gear, little discussed the effects of parabolic modifications on beveloid gear pair transmission with crossed axes.

In this paper, the generating procedure with parabolic modifications for the crossed beveloid gears with small shaft angle was proposed. And the analytical mesh model with misalignments of crossed beveloid gear transmission was developed. Then the effects of parabolic modifications on contact path and contact ratio was investigated. Also, the misalignments sensitivity was analyzed for the crossed beveloid gear pair with and without parabolic modifications.

2. Beveloid gear tooth profile with parabolic modification

Crossed beveloid gear pair is usually used to transmit motion and power with a small shaft angle which is less than 20° and the transmission diagram is shown in Fig. 1, the shortest distance of axials of two beveloid gears is defined as offset, and the angle between two axials is called shaft angle. Beveloid gear can be manufactured by conventional hobbing method, based on the generation concept of beveloid gear proposed by Mitome [20], the generation process of beveloid gear can be consider as the envelope process of rack cutter. The normal section of the rack cutter with parabolic modification is shown in Fig. 2. Without parabolic modification, the common normal section mainly consists of the straight edges and the fillet curves. In the figure, the previous rack cutter with straight lines is represented using the dashed line and the proposed rack cutter with parabolic modification is represented using the solid line. Three coordinates are created for the normal section and the origin of S_n is located to the middle point of tooth space on the standard pitch line of rack cutter. The origins of S_{al} and S_{ar} are located to the apex of the parabola at the left side and the right side, respectively. X_{al} and X_{ar} are tangent to the parabola. a_j is the parabola coefficient that can determine the amount of profile modification. l_j ($j=1$ for the pinion, $j=2$ for the gear) is the surface parameter of rack cutter. S_0 is the normal space width which is equal to $\pi m_n/2$. a_n is the normal pressure angle.

The coordinates of the points on the parabola can be represented in the coordinate system S_{ar} by

$$R_{ar}^{j} = \begin{bmatrix} x_{ar}^{j} \\ y_{ar}^{j} \\ z_{ar}^{j} \end{bmatrix} = \begin{bmatrix} l_j \\ a_j l_j^2 \\ 0 \end{bmatrix} \qquad (j = 1, 2) \tag{1}$$

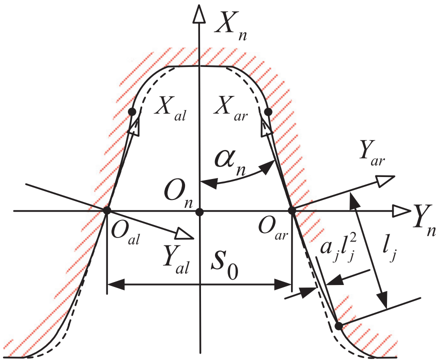
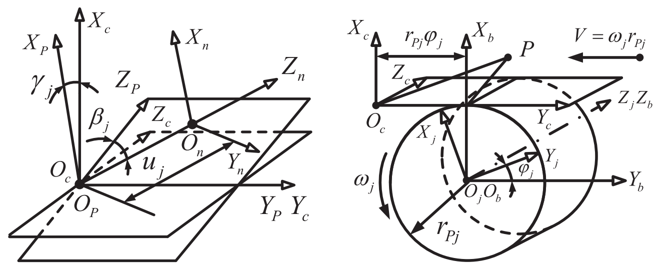


Fig. 2. Normal section of a rack cutter with parabolic modification.



(a) Conversion coordinates of rack cutter (b) Processing coordinates of beveloid gear

Fig. 3. Coordinate relationship for generating beveloid gear.

Through the coordinate transformation by $S_{ar} \rightarrow S_n$, the position vector can be represented in the coordinate system S_n by

$$R_{nr}^j = M_{na}^r R_{ar}^j \qquad (j = 1, 2) \tag{2}$$

Where the matric M_{na}^r is the transformation matrix as

$$M_{na}^r = \begin{bmatrix} \cos\alpha_n & \sin\alpha_n & 0 & 0 \\ -\sin\alpha_n & \cos\alpha_n & 0 & s_0/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Then, the modified beveloid gear can be generated using the coordinate relationship as shown in Fig. 3. β_j represents the helix angle on the pitch plane of rack cutter and γ_j represents the cone angle of beveloid gear. Through the series of transformation matrices given by $S_n \rightarrow S_P \rightarrow S_c$, the rack cutter surfaces can be represented in coordinate system S_c by

$$\begin{cases} x_c^j = \cos\gamma_j(a_j l_j^2 \sin\alpha_n + l_j \cos\alpha_n) + u_j \cos\beta_j \sin\gamma_j \mp \sin\beta_j \sin\gamma_j(a_j l_j^2 \cos\alpha_n - l_j \sin\alpha_n + s_0/2) \\ y_c^j = \pm\cos\beta_j(a_j l_j^2 \cos\alpha_n - l_j \sin\alpha_n + s_0/2) + u_j \sin\beta_j \\ z_c^j = u_j \cos\beta_j \cos\gamma_j - \sin\gamma_j(a_j l_j^2 \sin\alpha_n + l_j \cos\alpha_n) \mp \cos\gamma_j \sin\beta_j(a_j l_j^2 \cos\alpha_n - l_j \sin\alpha_n + s_0/2) \end{cases} \tag{4}$$

Where the upper sign of Eq. (4) indicates the right-side parabola, while the lower sign represents the left-side parabola.

The unit normal vector of two sides can be represented by

$$n_c^j = \frac{\partial R_c^j}{\partial l_j} \times \frac{\partial R_c^j}{\partial u_j} \Bigg/ \left| \frac{\partial R_c^j}{\partial l_j} \times \frac{\partial R_c^j}{\partial u_j} \right| = [n_{xc}^j, n_{yc}^j, n_{zc}^j]^T \tag{5}$$

In Fig. 3(b), S_j is the coordinate system attached to beveloid gear, S_b is the auxiliary coordinate system, r_{Pj} represents the reference pitch circle radius, and φ_j represents the gear rotation angle in the auxiliary coordinate system, Through the series of transformation matrices given by $S_c \rightarrow S_b \rightarrow S_j$, the mathematical model of beveloid gear tooth surface with parabolic

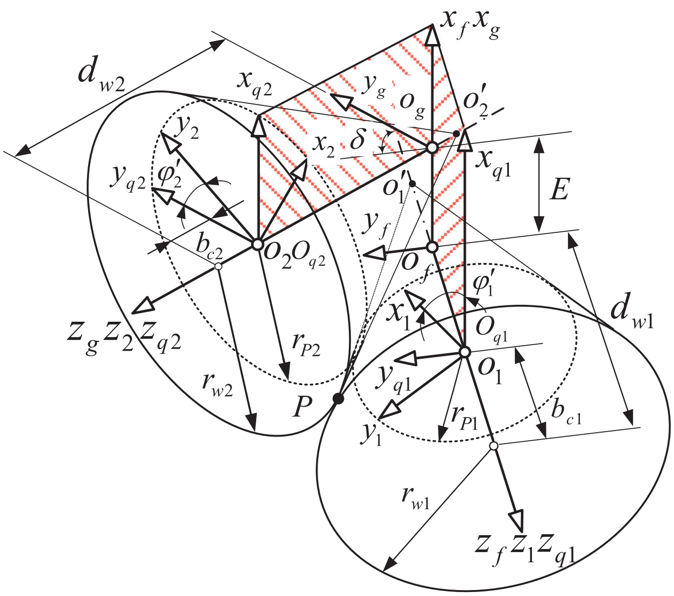


Fig. 4. The meshing model of crossed beveloid gears.

modification in coordinate system S_j can be represented by

$$\begin{cases} x_j = x_c^j \cos\varphi_j - y_c^j \sin\varphi_j + r_{Pj}(\cos\varphi_j + \varphi_j \sin\varphi_j) \\ y_j = x_c^j \sin\varphi_j + y_c^j \cos\varphi_j + r_{Pj}(\sin\varphi_j - \varphi_j \cos\varphi_j) \\ z_j = z_c^j \\ \varphi_j = (n_{xc}^j y_c^j - n_{yc}^j x_c^j)/(n_{xc}^j r_{Pj}) \end{cases} \tag{6}$$

Where φ_j is the rotating angle of beveloid gear, x_j, y_j, z_j represent the position vector of the point on the tooth surface of beveloid gear, r_{Pj} denotes the reference pitch circle radius.

3. Meshing model with misalignments

After the derivation of the mathematical model of beveloid gear tooth surface with parabolic modification, the analytical mesh model of crossed beveloid gear transmission was developed as shown in Fig. 4 [11]. δ is the shaft angle. E is the offset. Point P is the pitch point. r_{w1} and r_{w2} are the working pitch radius of the pinion and gear, respectively. And r_{P1} and r_{P2} are the reference pitch radius the pinion and gear, respectively. Four coordinate systems are attached to these model, S_1 and S_2 are moving coordinate systems attached to the pinion and gear, respectively. S_f and S_g are corresponding reference coordinate systems of S_1 and S_2. S_{q1} and S_{q2} are auxiliary coordinate systems which represent the initial positions of the pinion and gear, respectively. The roll angles of the pinion and gear relative to reference coordinate systemsare are φ_1' and φ_2', respectively. $d_{w1,\,2}$ is the distance between the origin point of reference coordinate system and the working pitch surface. $b_{c1,\,2}$ is the distance between the reference pitch surface and working pitch surface.

Since assembly errors are unavoidable in a practical application, it is necessary to investigate the sensitivity of the mesh behaviors for the crossed beveloid gear pair with parabolic modifications to misalignments. The misalignments are described in Fig. 5. Four coordinate systems denoted by S_e, S_δ, S_{d1} and S_{d2} are created to define the assembly errors. Δe which is the deviation between S_e and S_g represents the offset error which is the deviation of the shortest distance of two beveloid gears, $\Delta\delta$ which is the deviation between S_δ and S_f represents the shaft angle error which is the angular deviation of two beveloid gears, Δd_1 which is the deviation between S_{d1} and S_{q1} represents the axial position error of beveloid pinion, and Δd_2 which is the deviation between S_{d2} and S_{q2} represents the axial position error of beveloid gear, Δd_1 and Δd_2 means the distance that beveloid gears deviate from ideal location along their own axials. The signs of four assembly errors are positive in the Fig. 5.

As shown in the figure, S_f is the global coordinate system which is fixed to the frame. To perform the tooth contact analysis, the homogeneous surface coordinates and the normal vector of the beveloid pinion and gear should be transferred to the fixed coordinate system S_f and the position and normal vector can be represented by

$$R_f^j = M_{fj} R_j \tag{7}$$

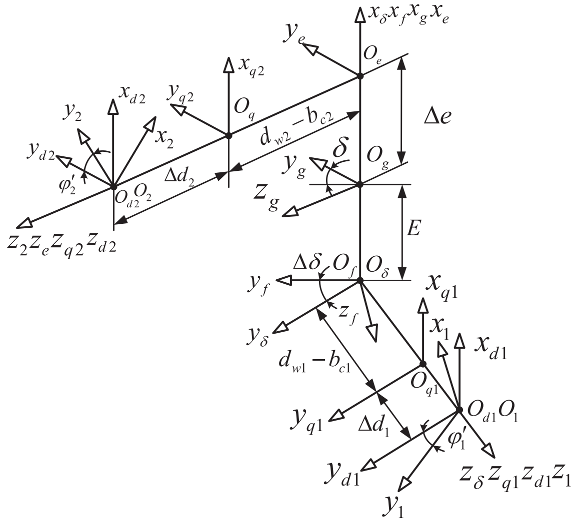


Fig. 5. Assembly errors of a crossed beveloid gear pair.

$$n_f^j = L_{fj} n_j \tag{8}$$

Where $R_f^j\,(j = 1, 2)$ is the position vector of the beveloid pinion and gear in coordinate system S_f. $n_f^j(j = 1, 2)$ is the unit normal vector of the beveloid pinion and gear in coordinate system S_f. The coordinate transformation matrixes for the beveloid pinion can be obtained as follow

$S_1(x_1, y_1, z_1) \rightarrow S_f(x_f, y_f, z_f)$

$$M_{f\delta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Delta\delta & -\sin\Delta\delta & 0 \\ 0 & \sin\Delta\delta & \cos\Delta\delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$M_{\delta q1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dw_1 - bc_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$M_{q1d1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$M_{d11} = \begin{bmatrix} \cos\varphi'_1 & -\sin\varphi'_1 & 0 & 0 \\ \sin\varphi'_1 & \cos\varphi'_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$M_{f1} = M_{f\delta} M_{\delta q1} M_{q1d1} M_{d11} \tag{13}$$

$$L_{f1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\delta & -\sin\Delta\delta \\ 0 & \sin\Delta\delta & \cos\Delta\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi'_1 & -\sin\varphi'_1 & 0 \\ \sin\varphi'_1 & \cos\varphi'_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

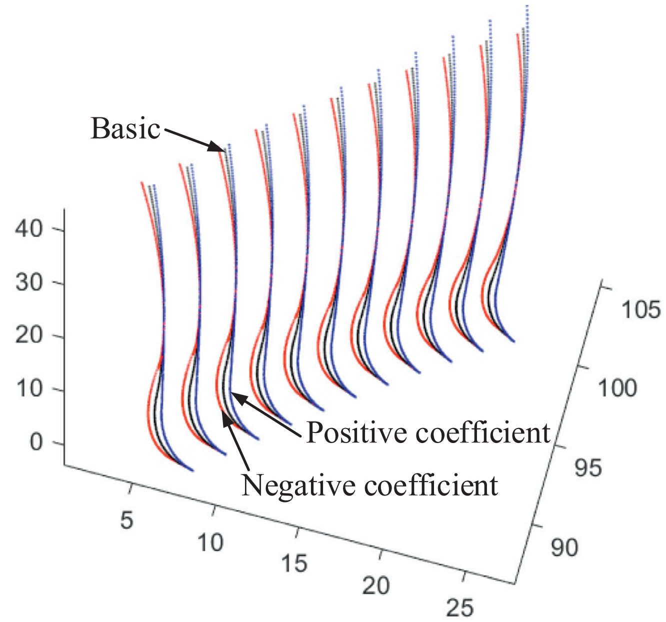


Fig. 6. The effect of the sign of parabola coefficient on beveloid gear tooth flank.

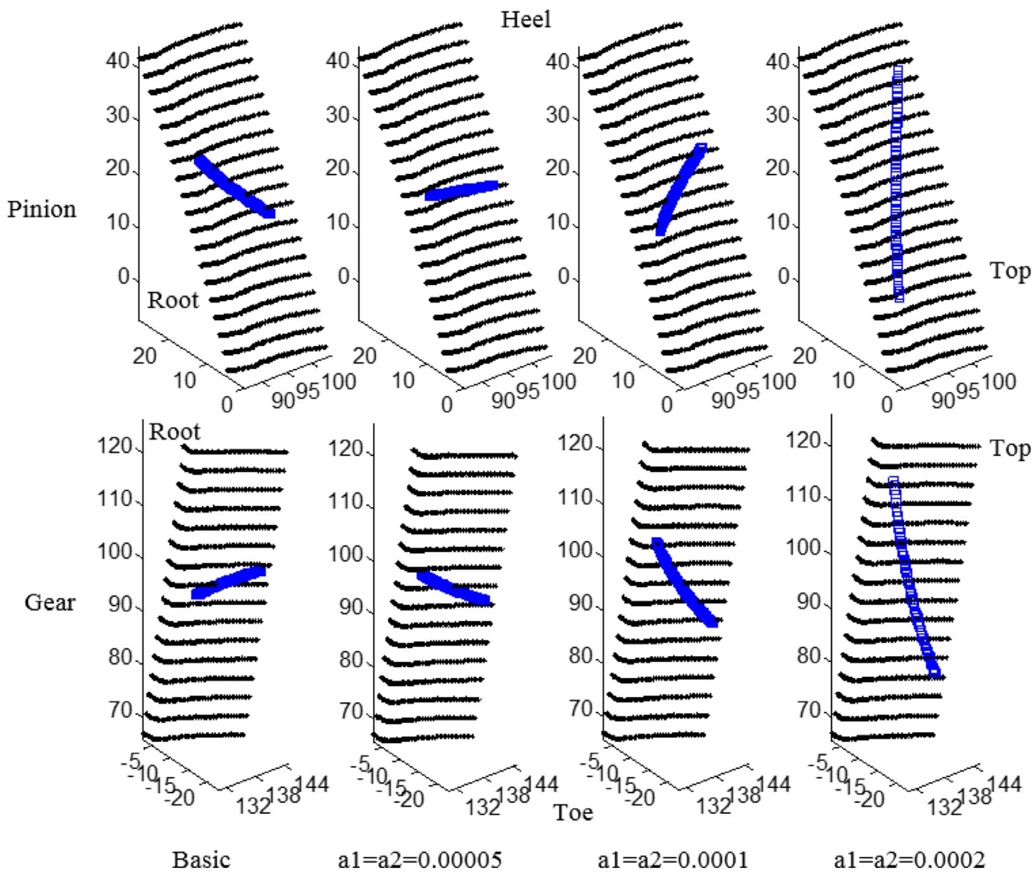


Fig. 7. Effect of positive parabola coefficients both for the pinion and gear on contact path.

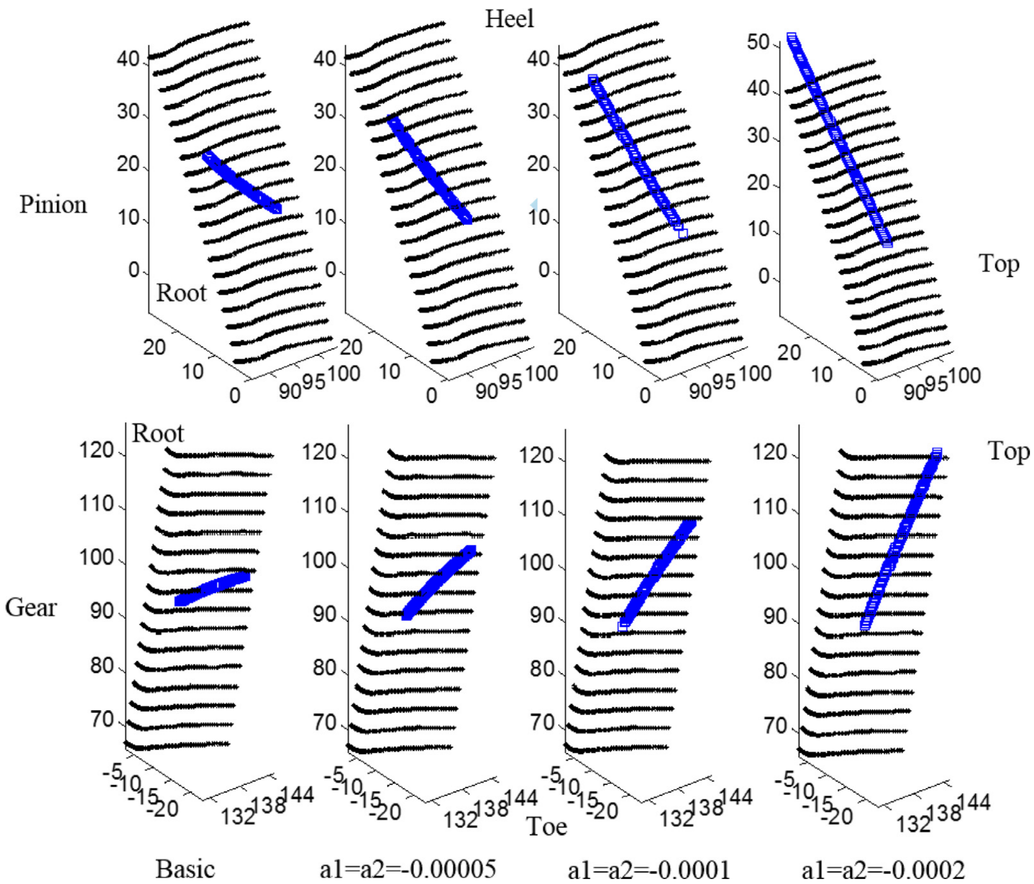


Fig. 8. Effect of negative parabola coefficients both for the pinion and gear on contact path.

Similarly, the transformation matrix M_{f2} and L_{f2} for the beveloid gear can be obtained. According to the meshing theory, the tooth surfaces of beveloid pinion and gear should be in continuous tangency. Therefore, they must have equal coordinates and normal vectors for the contact point. Then, the contact equation can be represented by

$$\begin{cases} R_f^1(l_1, u_1, \varphi'_1) = R_f^2(l_2, u_2, \varphi'_2) \\ n_f^1(l_1, u_1, \varphi'_1) = -n_f^2(l_2, u_2, \varphi'_2) \end{cases} \tag{15}$$

Since $n_f^j\ (j = 1, 2)$ is the unit vector, the above set of equations can be reduced to a system of 5 independent scalar algebraic equations. When the roll angle of beveloid pinion or gear is given, the set of equations can be solved.

4. Numerical examples

Using the proposed mathematical model of the tooth surface with parabolic modification and the analytical mesh model considering the misalignments, the effects of the sign (plus or minus) of parabola coefficient $a_j\ (j = 1, 2)$ on the contact path and contact ratio will be examined, as well as the effects of the misalignments. The geometry design parameters are listed in Table 1. According to the research of previous scholars [1,11], in the geometric design process of helix beveloid gear transmission with crossed axes, pinion always has a positive helix angle, while gear has a negative one, so the typical type of geometric parameters is discussed to investigate the effect of parabolic modification on crossed beveloid gear transmission.

4.1. Effects of parabolic modification parameters

Fig. 6 shows the effect of the sign of parabola coefficient on beveloid gear tooth flank, the gear parameters of this tooth flank is from the pinion in Table 1, the black line represents the tooth flank of basic involute beveloid gear, the blue one and the red one represent the tooth flank of parabolic modified beveloid gear with positive parabola coefficient and negative parabola coefficient, respectively. For convenience, the value of parabola coefficient is 0.01 mm^{-1} which is relatively large. According to Figs. 2 and 3, we can know that, when the parabola coefficient is positive, the tooth surface of beveloid gear

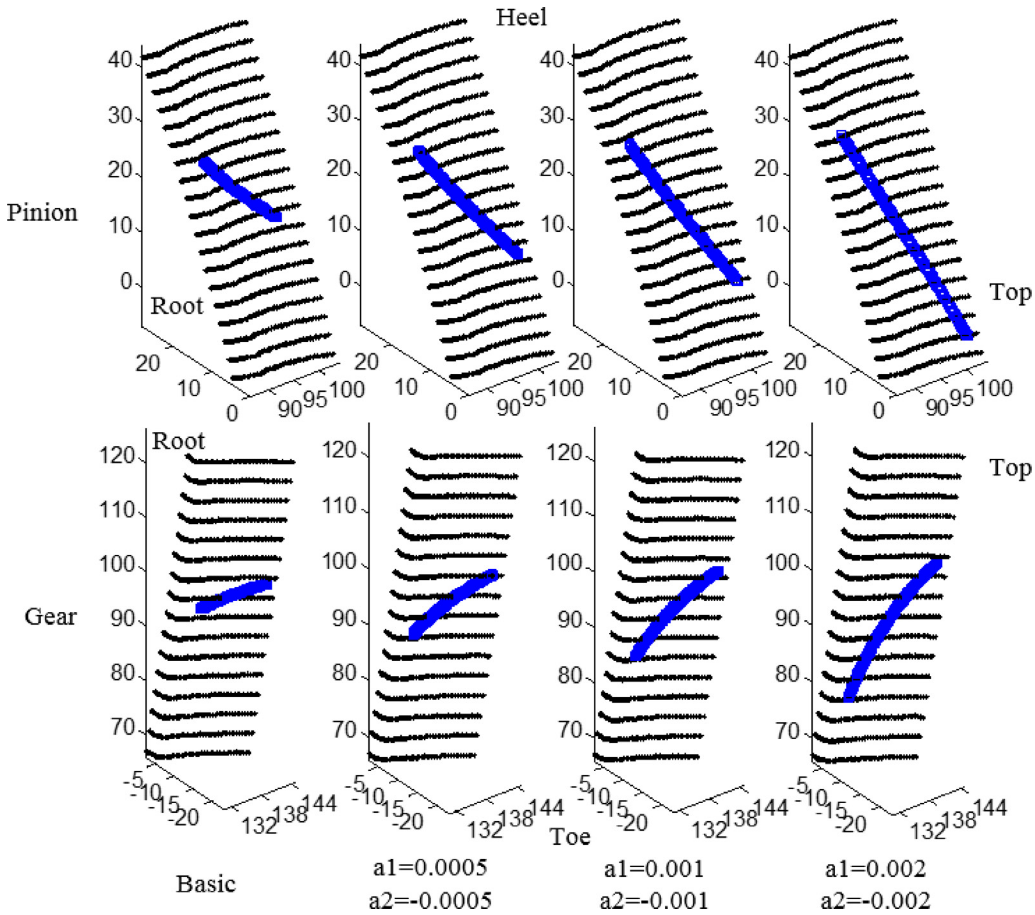


Fig. 9. Effect of positive pinion parabolic modification coefficient and negative for the gear on contact path.

which is generated by parabola rack cutter is concave relative to conventional involute beveloid gear, the blue one. When the parabola coefficient is negative, the tooth surface is convex relatively, the red one.

In this section, the effects of the parabolic modification parameters on the contact path and contact ratio will be investigated. The contact ratio of beveloid gear pair is a significant technical index and can be defined by

$$CR = TC/TZ \tag{16}$$

Where TC is the rolling angle for the pinion in one mesh cycle. $TZ = 360^\circ/N_1$ is the rolling angle for one circular pitch, N_1 is the tooth number of pinion.

In Figs. 7–10, the first row of each contact path figure shows the contact path on the tooth flank of pinion under parabolic modified, the left side of each tooth flank is the root side of beveloid gear, while the right side is top side, and the top of each tooth flank is the heel side of beveloid gear, while the bottom is the toe side. The second row of each contact path

Table 1
Gearing parameters for a pair of beveloid gear with parabolic modification.

Symbol	Pinion	Gear
Normal module (mm)	6	6
Normal pressure angle (°)	20	20
Number of teeth	29	45
Cone angle (°)	3.0	0.8
Helix angle (°)	25(RH)	−10.57(LH)
Offset (mm)	229	
Shaft angle (°)	15	
Profile shifting factor	0.17	0.2181
Total face-width (mm)	50	60

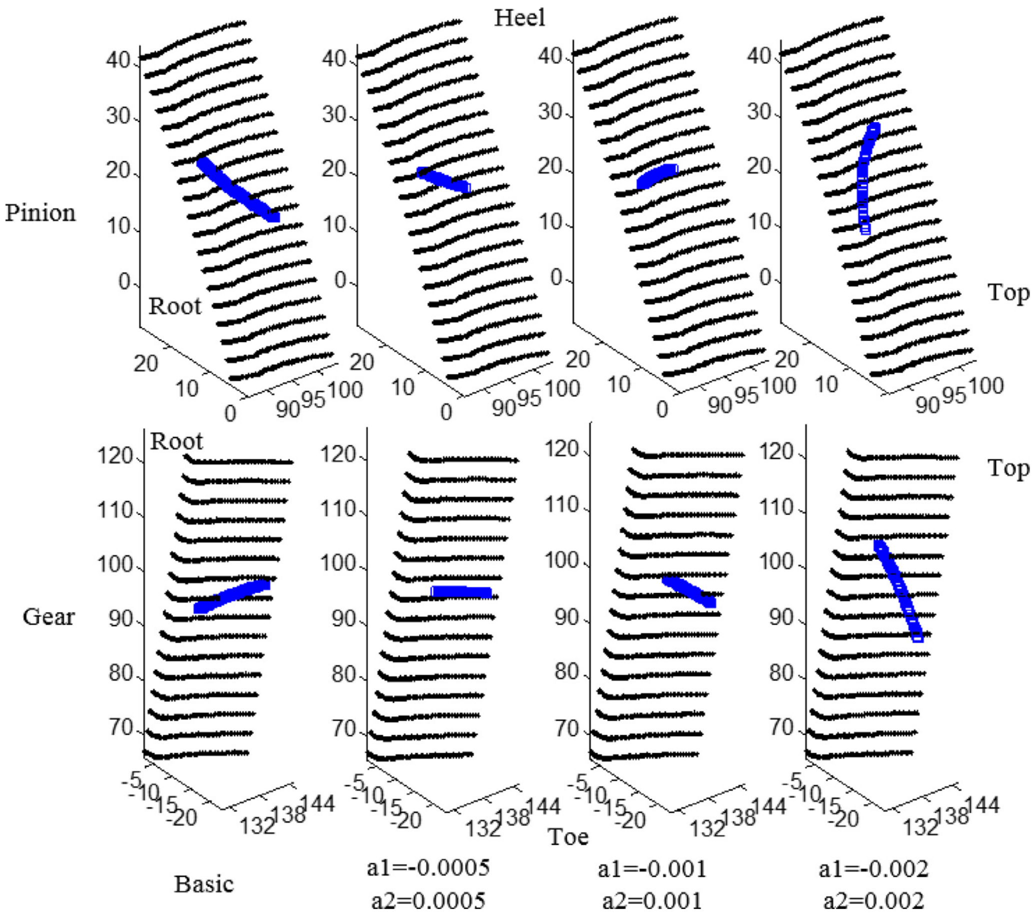


Fig. 10. Effect of negative pinion parabolic modification coefficient and positive for the gear on contact path.

figure shows the contact path on the tooth flank of gear under parabolic modified, the location of tooth flank of gear in the coordinate system is the same with pinion.

Case 1: Positive parabola coefficients both for the pinion and gear

When the parabolic modification coefficients of beveloid pinion and gear are both positive, the contact paths on the tooth surface for different parabola coefficients are shown in Fig. 7. The unit of parabola coefficient is millimeter. It can be seen that the direction of contact path changes gradually with the increase of the coefficient from 0 to 0.0002 mm^{-1} and the angle between contact path and axial direction is increased initially then reduced with opposite direction. Also, the length of contact path becomes longer.

Case 2: Negative parabola coefficients both for the pinion and gear

When parabola coefficients of pinion and gear are both negative, the contact paths on the tooth surface for different parabola coefficients are shown in Fig. 8. It can be seen that the length of contact path become longer and the angle between contact path and axial direction decreases obviously with the decrease of parabola coefficient from 0 to $-0.0002\,mm^{-1}$. Also the location of contact path moves from the middle to the heel side on the tooth surface. Furthermore, the location of contact path can be moved to the middle of tooth surface by optimizing the profile shifting factor of pinion and gear.

Case 3: Positive pinion parabolic modification coefficient and negative for the gear

With a positive parabola coefficient for the pinion and a negative value for the gear, the contact paths on the tooth surface for different parabola coefficients are shown in Fig. 9. As the parabola coefficient increases from 0 to 0.002 mm^{-1}, the length of contact path becomes longer and the angle between contact path and axial direction decreases slightly. Also the location of contact path of pinion and gear moves from middle to toe side with the increase of the parabola coefficient.

Case 4: Negative pinion parabolic modification coefficient and positive for the gear

With a negative parabola coefficient for the pinion and a positive value for the gear, the contact paths on the tooth surface for different parabola coefficients are shown in Fig. 10. From the results, the length of contact path becomes shorter initially then longer with the increase of the parabola coefficient from 0 to 0.002 mm^{-1}. And the angle between contact path and axial direction is increased initially then reduced with opposite direction.

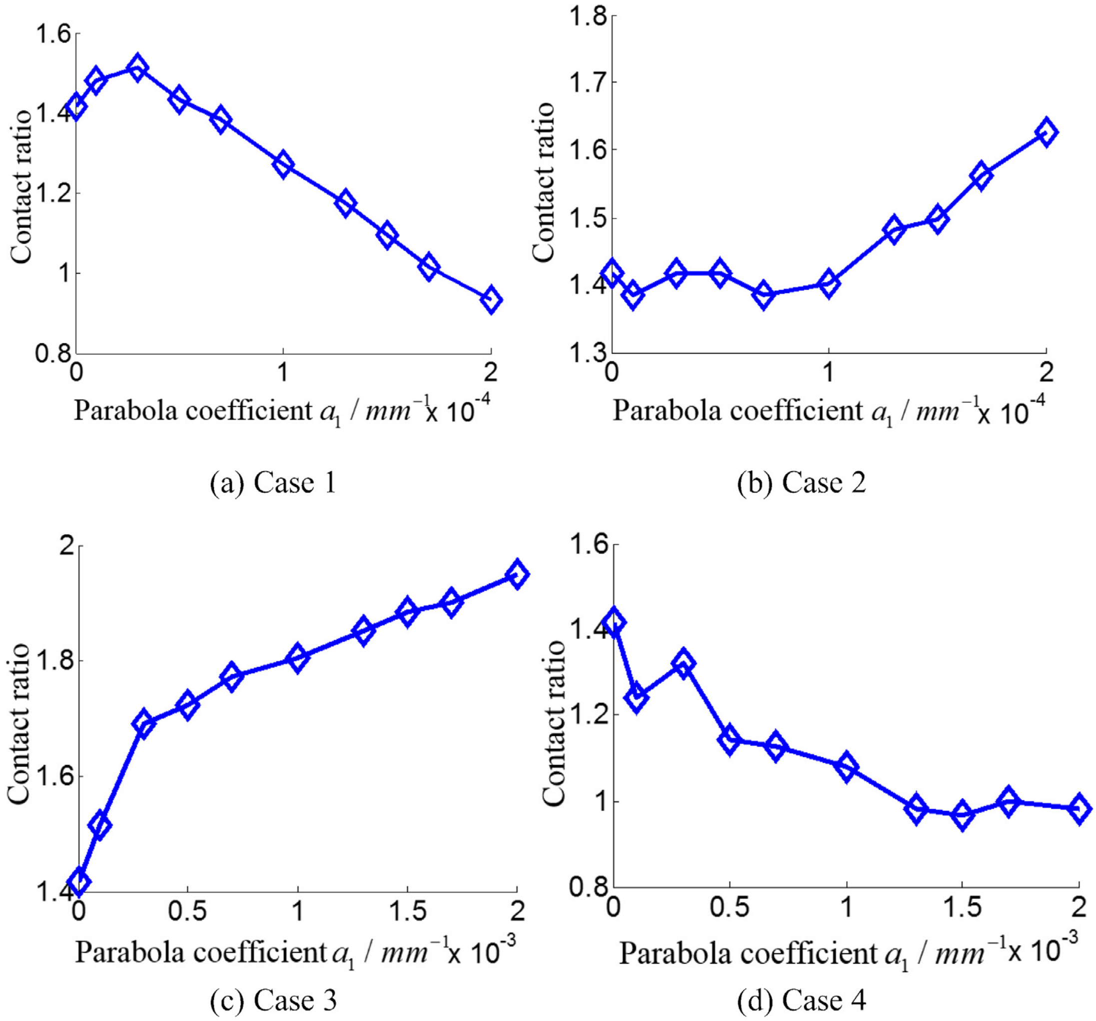


Fig. 11. Contact ratio of modified beveloid gear pair.

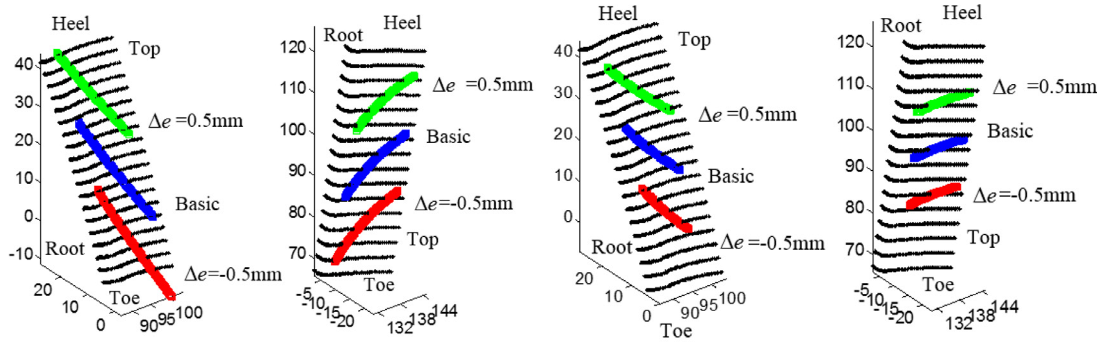
Based on the results shown above, the angle between the contact path and the axial direction is more sensitive for Cases 1 and 4. That mean the relative sliding between the engaged pinion and gear is easier to be controlled for Cases 2 and 3. However, compared with Cases 2 and 3, the precision for the cutter parabola coefficient is much too high for Case 2. So the parabola coefficient in Case 3 is suggested for the design of the cutter.

The effects of parabola coefficients on the contact ratio are shown in Fig. 11. It can be seen that when parabola coefficients are both positive (Case 1), a small parabola coefficient can increase the contact ratio, but the contact ratio decreases obviously with the increase of parabola coefficient. In Fig. 11(b), when parabola coefficients are both negative (Case 2), a small value of parabola coefficient (Approximately less than 0.0001 mm^{-1}) has little effect on contact ratio. However, when the value increase to a certain stage (Approximately larger than 0.0001 mm^{-1}), the contact ratio increases obviously.

In Fig. 11(c), parabola coefficient of pinion is positive while gear is negative (Case 3), the increase of parabola coefficient tends to increase the contact ratio obviously. In Fig. 11(d), parabola coefficient of pinion is negative while gear is positive (Case 4), it can be seen that the increase of parabola coefficient tends to decrease the contact ratio of modified beveloid gear pair obviously. From the analysis, a conclusion can be obtained that the design situation with a positive parabola coefficient for the pinion and a negative parabola coefficient for the gear is better that other combinations of the parabola coefficients.

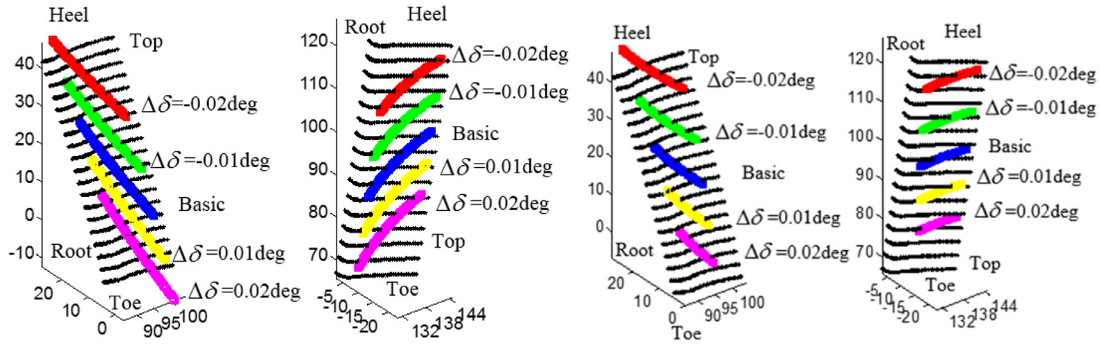
4.2. Misalignment sensitivity analysis

Base on the analysis in Section 4.1, the parabola coefficient $a1 = 0.001\ mm^{-1}$, $a2 = -0.001\ mm^{-1}$ in Case 3 was used to investigate the misalignments effects subsequently. the effects of misalignments on the contact paths of beveloid gear pair with and without parabolic modifications are shown in Figs. 12–15. For each specific figure, the left tooth surface represents the pinion surface and the right tooth surface represents the gear surface. The effect of offset error on the contact path is



(a) With parabolic modification (b) Without parabolic modification

Fig. 12. Contact path of beveloid gear pair under offset error.



(a) With parabolic modification (b) Without parabolic modification

Fig. 13. Contact path of beveloid gear pair under shaft angle error.

shown in Fig. 12. For the offset error −0.5 mm and 0.5 mm, the location of the contact path moved to the toe and heel, respectively. And the contact path of parabolic modified beveloid gear pair is longer than conventional involute beveloid gear pair. Due to the existence of the offset error, the deviation value of contact path of parabolic modified beveloid gear pair is smaller than the conventional involute beveloid gear pair.

The effect of shaft angle error on the contact path is shown in Fig. 13. Opposite to the effect of the offset error, the positive shaft angle error tends to move the location of the contact path to the toe, while the negative shaft angle error tends to move the location to the heel. Also, compared with the beveloid gear pair with and without parabolic modification, the contact path becomes longer and the deviation value of contact path from the base case becomes smaller due the existence of the offset error. In comparison, contact path of beveloid gear pair with parabolic modification is less sensitivity to shaft angle error than the conventional involute beveloid gear pair.

The effects of the axial position errors of pinion and gear on the contact paths are shown in Figs. 14 and 15. From the results, it can be seen that both the axial position errors of pinion and gear have little effects on contact paths.

5. Conclusions

A numerical approach based on exact beveloid gear geometry with parabolic modifications is applied to perform the tooth contact analysis. Then the effects of different parabolic modification parameters on the mesh behaviors were investigated. Also, the effects of four types of assembly errors on the contact of a pair of crossed beveloid gears were studied. From the results, the following specific conclusions can be obtained.

(1) The design situation with a positive parabola coefficient for the pinion and a negative parabola coefficient for the gear is strongly recommended due to a better contact path distribution and contact ratio than other combinations of the parabola coefficients.

(2) Both the offset error and shaft angle error tend to deviate the contact path from the theoretical middle region. Positive offset error and negative shaft angle error tend to shift the contact path to the heel, while negative offset error and positive shaft angle error tend to shift the contact path to the toe. The parabolic modifications can reduce the sensitivity of contact path to the offset error and shaft angle error for crossed beveloid gear transmission effectively.

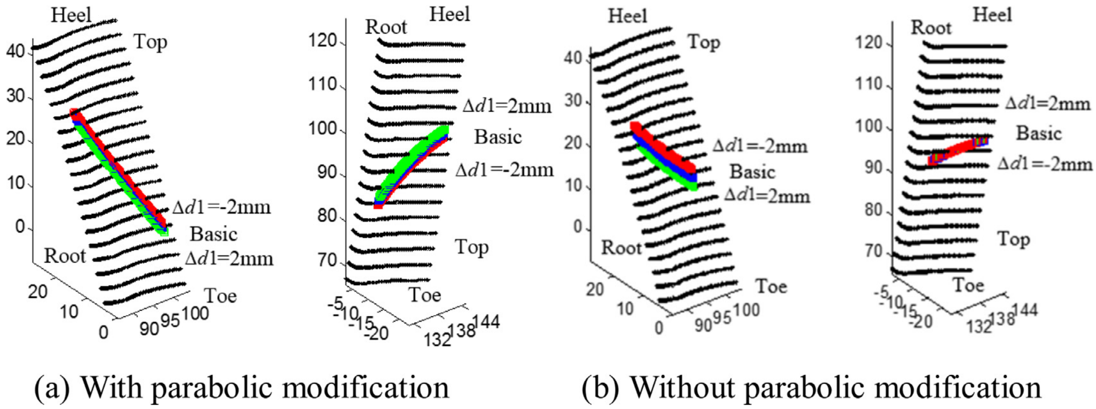


Fig. 14. Contact path of beveloid gear pair under axial position error of pinion.

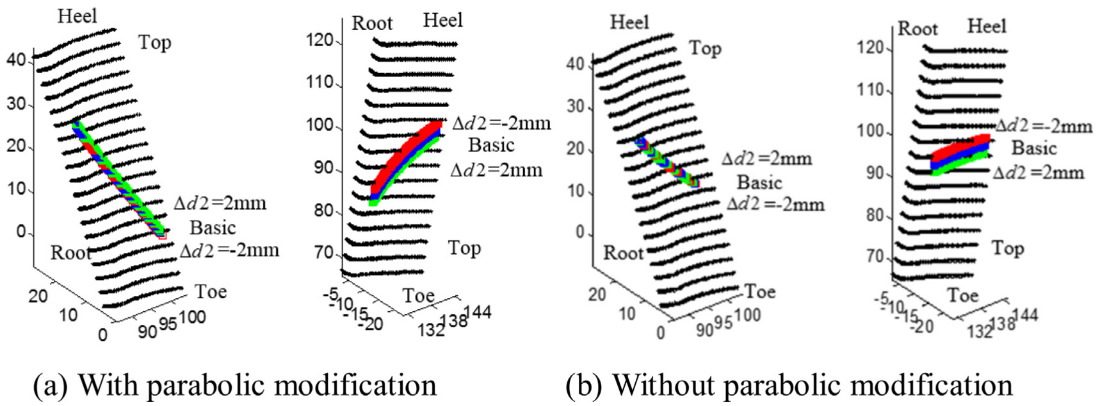


Fig. 15. Contact path of beveloid gear pair under axial position error of gear.

(3) Both axial pinion and gear position error has little influence on the mesh behaviors for the crossed beveloid gear pair with and without parabolic modifications.

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